FREQUENCY SPECTRA OF TURBULENT PULSATIONS AND LAMINAR BOUNDARY-LAYER STABILITY

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UDC 532.517.4

The characteristic frequencies of turbulent pulsations in a bounded viscous fluid flow are examined analytically.

It is shown in [1] that the most promising direction in the creation of a theory of turbulence is the study of the stream pulsating parameters. Special attention is paid here to the mutual relationship between the turbulent pulsations and the acoustic perturbations.

It is known from experimental investigations that there are characteristic maximum in the pulsations spectra in the boundary layer [2] and that the boundary layer is reponsive to external acoustic action in a definite frequency band [3]. These phenomena are explained by the existence of a quasiordered stream structure—systems of large-scale discrete vortices. Associated with the origination of such vortices is the transition from the laminar to the turbulent modes of motion. However, no theory connecting the stability of fluid motion, the integral and pulsating turbulent flow characteristics and their quasiordered structure has yet been developed.

The effect of long-range turbulent pulsations, the emission of oscillations by a discrete vortex which will propagate in all directions including along the normal to the streamlines, is examined in [4]. The physical nature of such waves has not yet been explained fully. When the fluid flow is bounded by a solid wall it is necessary to consider resonance interaction between the oscillations source and its mirror image relative to the wall, an imagninary source. A quasistationary pulsating field occurs in the presence of a set of oscillation sources in the stream. The characteristic (resonance) frequency of the oscillations at a point of the stream is here related to the distance from the wall.

The process of turbulent vortex development includes their nucleation on the outer boundary of the viscous sublayer, removal from the wall under the effect of Zhukovskii lift force, and growth of the linear dimensions associated with the pairwise merger of the vortices. A sufficiently strict determination of the vortex dimensions, their lifetime, and the characteristic frequency of the pulsations [5] permits the assumption that this process is a stage of the self-oscillation process in which feedback is accomplished by the long-range waves.

Therefore, in the subsequent computations we shall start from the assumption that the growth and development of turbulent vortices are interrelated with the pulsating field, while the zone of quasiordered fluid motion agrees with the resonance zone.

In the near-wall domain the resonance zone is bounded by the value of the maximum turbulent pulsation frequency and in the outer boundary-layer domain by long-range wave damping. Special experiments are necessary for an accurate determination of these limits. As yet, we shall start from the assertion in [6] that quasiordered structures induce a fundamental contribution in the production of turbulent tangential stresses. Then there follows from similarity considerations that the distance from the wall to the lower boundary of the resonance zone is proportional to the viscous sublayer thickness, and to the upper boundary is proportional to the turbulent boundary layer thickness, i.e.,

$$k_1 \frac{\nu}{u_*} \leqslant y \leqslant k_2 \delta. \tag{1}$$

The transition to a state of quasiordered motion should be related to a qualitative change in the magnitude of the tubulent tangential stress. In this connection we select the point of the beginning of the logarithmic section of the velocity profile $y^+ = 30$ as the lower boundary. Then $k_1 = 30$.

Odessa Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 6, pp. 900-905, June, 1984. Original article submitted January 25, 1983.

The velocity profile in the outer domain of a turbulent boundary layer deviates "upward" from the logarithmic curve and the miscibility factor diminishes. However, these facts do not permit exact determination of the upper boundary of the resonance zone. For a numerical example we select the value $k_2 = 0.6$ by noting the stability of this boundary.

Starting from the proposed model, we determine the boundary of the turbulent pulsation frequency band. We shall consider the initial angular velocity of the vortex ω to be determined by the gradient of the average velocity at the point of formation $\omega = 1/2 \cdot d\bar{u}/dy$. The frequency of the oscillations radiated by a nonsymmetric rotating vortex is $f = \omega/2\pi$.

The gradient of the average velocity reaches the maximum value at the wall. Therefore, the vortices being formed at the wall will radiate oscillations at the maximal frequency

$$f_{max} = \frac{1}{4\pi} \left| \frac{d\bar{u}}{dy} \right|_{y=0} = \frac{1}{4\pi} \frac{u_*^2}{\nu}, \tag{2}$$

where f_{max} is the upper bound of the turbulent pulsation frequency band.

In a first approximation we assume the propagation velocity of the oscillations in the transverse direction a to be independent of the distance from the wall. Then the frequencies $f_n = na/2y$, where $n = 1, 2, 3, \ldots$, will be resonant for oscillation sources that are a distance y from the wall. Since the wave damping factor grows in proportion to the square of the frequency [7], only the least resonance frequency (the first harmonic) will be of practical value

$$f_1 = a/2y. \tag{3}$$

Equating (2) and (3) and substituting the least resonance distance from (1), we obtain a formula to determine the propagation velocity of the oscillations

$$a = \frac{k_1}{2\pi} u_*. \tag{4}$$

In turbulent flows the quantity a always turns out to be less than the characteristic stream velocity, i.e., according to the terminology of [8] this is the propagation velocity of pseudosubsonic oscillations.

The minimal frequency pulsations interact with vortices on the outer boundary of the resonance zone (1). Taking account of (3) and (4) we obtain

$$f_{min} = \frac{k_1}{4\pi k_2} \frac{u_*}{\delta}. \tag{5}$$

Using the formulas obtained and the known semiempirical relationships [9], we compute the boundary layer on a semiinfinite smooth plate. In this case the dynamical velocity is

$$u_* = \sqrt{c_f/2}\, u_\infty. \tag{6}$$

From (2) and (6), the Strouhal number corresponding to the maximal pulsation frequency is

$$\mathrm{Sh}_{max} = \frac{f_{max} x}{u_\infty} = \frac{c_f \mathrm{Re}}{8\pi}. \tag{7}$$

The coefficient of local friction for a laminar boundary layer is $c_f^l = 0.664\mathrm{Re}^{-0.5}$. In this case the maximum dimensionless pulsation frequency is

$$\mathrm{Sh}_{max}^l = 2.64 \cdot 10^{-2} \cdot \mathrm{Re}^{0.5}. \tag{8}$$

According to the Prandtl theory, for a turbulent boundary layer

$$c_f^T = 5.92 \cdot 10^{-2} \cdot \mathrm{Re}^{-0.2}. \tag{9}$$

Then the upper bound of the frequency band is from (7)

$$\mathrm{Sh}_{max}^T = 2.36 \cdot 10^{-3} \cdot \mathrm{Re}^{0.8}. \tag{10}$$

The dependences (8) and (10) are, respectively, represented by the curves 1 and 2 in the figure.

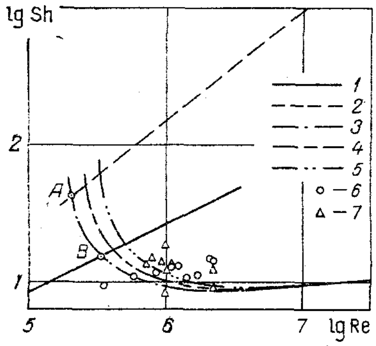

Fig. 1. Dependence of the dimensionless turbulent pulsation frequencies Sh on the Reynolds number Re for flow around a smooth plate. Computation: 1) using (8); 2) using (10); 3-5) using (13) for $Re_{cr} = 3.2 \cdot 10^5$, and $5 \cdot 10^5$. Experiment: 6) from data in [2]; 7) from [3].

From (5) and (6) the Strouhal number corresponding to the minimum turbulent pulsation frequency is

$$\mathrm{Sh}_{\min} = \frac{f_{\min} x}{u_\infty} = \frac{k_1}{4\pi k_2} \frac{x}{\delta} \sqrt{\frac{c_f^{\mathrm{T}}}{2}}. \tag{11}$$

In this case δ is the turbulent boundary-layer thickness which does not start from the plate leading edge. It can be determined from the relationship

$$\frac{\delta}{x} = \frac{0{,}38}{\mathrm{Re}^{0.2}} - \frac{D}{\mathrm{Re}}. \tag{12}$$

The coefficient D depends on the critical Reynolds number. Thus, D = 5800 for $Re_{cr} = 3.2 \cdot 10^5$.

Substituting the values of c_f^T and δ from (9) and (12) and the numerical values of the coefficients k_1 and k_2 into (11), we obtain the following expression for the lower boundary of the turbulent pulsation spectrum

$$\mathrm{Sh}_{\min} = \frac{0{,}685\,\mathrm{Re}^{0.9}}{0.38\,\mathrm{Re}^{0.8} - D}. \tag{13}$$

Graphs of the dependence (13) for values of Re_{cr} equal to $3.2 \cdot 10^5$, $4 \cdot 10^5$, and $5 \cdot 10^5$ are shown by curves 3, 4, and 5, respectively. Curve 3 intersects the lines 1 and 2 at the points B and A, respectively.

It is seen from the figure that for $Re < Re_A$ the single possible mode of motion is laminar. The pulsations originating here have a frequency that does not exceed Sh_{max}^l and damp out rapidly. To the right of the point A pulsations in the frequency range between Sh_{min} and Sh_{max}^T are possible in principle, but for $Re < Re_B$ the energy of the stream itself is inadequate to maintain such oscillations. The velocity profile in which $\delta^+ = 30$ would correspond to the turbulent flow characterized by the point A in the figure. Such a velocity distribution is impossible in actually existent turbulent flows. In a turbulent boundary layer $\delta^+ = 194$ for the low value of the critical Reynolds number $Re_{cr} = 3.2 \cdot 10^5$.

And, finally, for $Re > Re_B$ the velocity gradient at the wall is adequate for pulsations at a "resonating" frequency to occur. If the vortex natural oscillations enter into resonance with a quasistationary pulsating field, then the growth of turbulent pulsations starts in the boundary layer, the velocity diagram changes, and the velocity gradient at the wall increases. Consequently, the laminar boundary layer goes over into a turbulent layer with pulsations in the range from Sh_{min} to Sh_{max}^T.

It should be noted that for this selection of the coefficients k_1 and k_2 the coordinates of the intersections of curves 3, 4, and 5 (Fig. 1), constructed for different values of Re_{cr}, with the line 1 are quite close to the given values of Re_{cr}. Small discrepancies can be explained by the approximate nature of the semiempirical formulas used. Such agreement evidently objectively reflects a substantial interaction between the pulsation spectra and the transition from the laminar to the turbulent motion regimes.

As was noted in [5], experimental confirmation of the relations obtained can be executed by both direct and indirect methods. The direct method is a experimental study of the turbulent pulsation spectra. Results of investigating the energy spectra of longitudinal velocity pulsations in the boundary layer on a plate are presented in [2]. It is shown that such spectra have several maximums, where the specific fraction of each of the maximums varies with the change in distance from the wall y^+. The dimensionless pulsation frequencies Sh that correspond to the principal energy maximum are shown by the points 6 in the figure.

Equivalent values of the number Re were calculated from (9) in this case, from experimentally determined values of c_f^T.

The indirect method is to determine the boundary layer reaction to external acoustic perturbations of different frequencies. Points 7 at which the maximum growth in pulsations is observed, according to [3], under the effect of an external acoustic field, are also presented in the figure.

As is seen from the figure, the experimental points are lumped in the band between Sh_{min} and Sh_{max}^T, near its lower boundary. It can then be concluded that generation of "long-range waves" occurs most intensively in the outer turbulent boundary layer region, which is related, in all probability, to destruction of the discrete vortices. Experimental and theoretical investigations of the transients within a discrete vortex are needed for clarification of the question of the nature of such waves.

Therefore, the assumption that the origination of turbulence is associated with resonance phenomena permits a sufficiently accurate determination, despite a number of simplifying assumptions, of the characteristic turbulent pulsation frequencies and an explanation of the transition from the laminar to the turbulent motion regimes.

NOTATION

x and y, longitudinal and transverse coordinates; δ, boundary-layer thickness on the plate; $\bar{u}$, average velocity at a point of the stream; u_*, dynamical velocity; u_∞, free stream velocity far from the plate; a, propagation velocity of the oscillations; ν, fluid kinematic viscosity; ω, vortex angular velocity; f, pulsation frequency; c_f, local friction coefficient; k_1, k_2, D, dimensionless coefficients; $y^+ = u_* y/\nu$, dimensionless distance from the wall; $\delta^+ = u_*\delta/\nu$, dimensionless boundary-layer thickness; $Re = u_\infty x/\nu$, Reynolds number; $Sh = fx/u_\infty$, Strouhal number. Superscripts: l, refers to the laminar and T to the turbulent boundary layer.

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